

Fig. 4 Pitot traverse between model and shock wave

graphs is shown on Fig. 4 and corresponds closely with the region of constant pitot pressure

The position of the internally mounted pitot is accurate to  $\pm 0.001$  in. and that of the externally mounted tube to  $\pm 0.02$  in. The pressures are measured to an accuracy of  $\pm 0.1$  psi

It has been shown that it is possible to measure the pitot pressure in the boundary layer on a model in the running time of a hypersonic gun tunnel (40 msec). Other results, which call for further investigation, are the identification of the sonic line in the boundary layer and the boundary of entropy layer

#### References

- Cox, R. N. and Winter, D. F. T., "Hypersonic flow," *Proceedings of the 11th Symposium of the Colston Research Society* (Royal Armament Research and Development Establishment, Bristol, 1959), pp. 111-141.
- Wood, N. B., "Nose bluntness effects on cone pressure and shock shape at  $M = 8.5$  to  $12.9$ ," *AIAA J.* 1, 1929-1930 (1963).

## Analysis of Air Arc-Tunnel Heat-Transfer Data

DANIEL E. ROSNER\*

AeroChem Research Laboratories, Inc., Princeton, N. J.

#### Nomenclature

- $c_p$  = specific heat of mixture  
 $D_i$  = Fick coefficient for diffusion of species  $i$  through the mixture  
 $h$  = specific enthalpy (including chemical contributions)  
 $k$  = Boltzmann constant  
 $k_R(T)$  = termolecular (homogeneous) atom recombination rate constant at temperature  $T$  ( $dn_i/dt = -2k_R n_i^2 n$ )  
 $k_{w,i}$  =  $(m_i n_{i,w})^{1-n} [k T_w / (2\pi m_i)]^{1/2} \gamma_i$   
 $Le_i$  = Lewis number  $\equiv D_i / [\lambda_f / (\rho c_p f)]$   
 $m_i$  = mass of species  $i$   
 $n$  = true reaction order  
 $n$  = number density ( $p/kT$ )  
 $N$  = number of thermochemical energy carriers  
 $p$  = pressure  
 $Pr$  = Prandtl number for heat conduction  $\equiv (\mu/\rho) / [\lambda_f / (\rho c_p f)]$   
 $Pr_D$  = Prandtl number for diffusion  $\equiv (\mu/\rho) / D_i$   
 $\dot{q}''$  = calorimeter heat flux

- $r_{Di}$  = recovery factor for chemical energy<sup>3</sup> associated with species  $i$ ;  $r_{Di} = (Le_i)^0$  for stagnation-point boundary layer  
 $St_{Di}$  = local Stanton number for convective transport of species  $i$   
 $T$  = absolute temperature  
 $u$  = component of gas velocity parallel to surface  
 $z^{(G)}$  = generalized gas-phase recombination parameter Eq. (4)  
 $z_i^{(W)}$  =  $k_{w,i} \rho_w (\rho_w \alpha_i)^{n-1} / (\rho u St_{Di})$   
 $\alpha_i$  = mass fraction of species  $i$  in the mixture  
 $\gamma$  = heterogeneous recombination coefficient (probability that an atom incident upon the surface will recombine)  
 $\lambda$  = thermal conductivity of gas mixture  
 $\mu$  = dynamic viscosity of mixture  
 $\rho$  = mass density of mixture  
 $\phi_i$  = extent of recombination for species  $i$ ;  $\phi_i = \Delta h_{chem,i} / \Delta h_{hem,i,max}$   
 $\Delta$  = operator meaning change in (across boundary layer)

#### Subscripts

- chem = chemical contribution  
 $e$  = at outer edge of boundary layer  
 $eq$  = pertaining to local thermochemical equilibrium  
 $f$  = chemically frozen  
 $i$  = pertaining to species  $i$   
 $min$  = minimum value (no recombination)  
 $max$  = maximum value (complete recombination)  
 $w$  = at wall  
 $1$  = pertaining to atoms in a binary mixture

#### Introduction

WETHERN<sup>1</sup> has compared data on subsonic air arc-tunnel heat transfer to a water-cooled copper calorimeter with the predictions of several limiting theoretical models and concluded that the assumptions of chemically frozen boundary-layer flow and a noncatalytic wall were virtually exact for most of the conditions studied. Since a copper surface can behave this way at stagnation pressures and enthalpies approaching 1 atm and  $10^4$  Btu/lbm, respectively, calorimeter catalytic activity becomes an important parameter, particularly if effective heats of ablation or total enthalpies are subsequently to be determined from heat-transfer data obtained at thermally significant dissociation levels. In this note, a direct procedure for estimating catalytic activities from such calorimeter heat-transfer data is illustrated, and a prediction is made of the stagnation pressure levels at which gas-phase atom recombination should mask the observed effects of calorimeter surface specificity under arc-tunnel conditions. Implications of this result for materials testing, and factors governing the probability of atom recombination on metal and metal oxide surfaces, are briefly discussed in the light of recent kinetic observations.

#### General Theory

It is a short step to go from limiting heat fluxes (i.e., predictions or data giving  $\dot{q}''_{max}$  and  $\dot{q}''_{min}$ ) and calorimeter heat flux data of the type presented in Ref. 1 to quantitative estimates of the relevant atom recombination probabilities, particularly when the stagnation pressure level and calorimeter size are sufficiently small so that gas-phase atom recombination (in the boundary layer) can be neglected.† Applying the methods exploited in Refs. 2 and 3, one can readily show that

$$\frac{\dot{q}'' - \dot{q}''_{min}}{\dot{q}''_{max} - \dot{q}''_{min}} = \frac{\sum_{i=1}^N \phi_i r_{Di} \Delta h_{chem,i,max}}{\sum_{i=1}^N r_{Di} \Delta h_{hem,i,max}} \quad (1)$$

† See section entitled "Gas-Phase Recombination."

Received January 16, 1964. This work was supported by the Propulsion Division of the Office of Aerospace Research, U. S. Air Force Office of Scientific Research, under Contract AF 49 (638) 1138. This paper is a revised version of AeroChem TP-76 (January 2, 1964).

\* Aeronautical Research Scientist. Associate Fellow Member AIAA.

As indicated, each species  $i$  that can transport thermochemical energy to the solid surface makes a contribution to the normalized difference between the observed heat flux  $\dot{q}''$  and the minimum (no recombination) heat flux  $\dot{q}''_{\min}$  in accord with the corresponding extent of reaction  $\phi_i$ , recovery factor  $r_{D,i}$ , and maximum possible chemical enthalpy change  $\Delta h_{\text{chem } i \text{ max}}$  across the boundary layer

The terms  $r_{D,i}$  and  $\Delta h_{\text{chem } i \text{ max}}$  can usually be evaluated from transport<sup>†</sup> and thermodynamic property considerations alone; however, both chemical kinetic and aerodynamic parameters enter into the determination of each  $\phi_i$  through the dimensionless grouping  $z_i^{(w)}$  defined in the Nomenclature. Indeed, it is an explicit knowledge of the  $\phi_i(z_i^{(w)})$  relation which, in principle, enables each recombination coefficient  $\gamma_i$  to be extracted from arcjet data and from predictions of the type reported in Ref. 1. For example, if 1) each heterogeneous recombination process proceeds independently of the others and is kinetically first order ( $n = 1$ ), 2) the surface temperature is sufficiently low so that  $\alpha_{i \text{ eq}}(T_w; p) \ll \alpha_i$ , and 3) gas-phase atom recombination<sup>§</sup> is negligible, then

$$\phi_i = z_i^{(w)} [1 + z_i^{(w)}]^{-1} \quad (2)$$

Although one could now formally determine each of the  $N$  values of  $\gamma_i$  from  $N$  or more calorimeter heat flux measurements in a partially dissociated gas mixture, as in any multi-parameter "fitting" procedure, the accuracy and precision demands on the data would be very great, particularly for  $N \geq 2$ . Because of this limitation and for purposes of illustration, consider the simple case of a partially dissociated diatomic gas. Then a single coefficient  $\gamma$  characterizes the kinetics of the gas/solid interaction, and the right-hand side of Eq. (1) simplifies to  $\phi$  (regardless of the magnitudes of  $r_D$  and  $\Delta h_{\text{chem } i \text{ max}}$ ). Making use of Eq. (2) and the definition of the catalytic parameter  $z^{(w)}$ , it follows that, for each heat flux measurement, one can compute<sup>¶</sup> a corresponding recombination coefficient  $\gamma$  from<sup>\*\*</sup>

$$\gamma = \left( \frac{2\pi m_i}{kT_w} \right)^{1/2} \left( \frac{\rho_e u_e St_D}{\rho_w} \right) \left( \frac{\dot{q}'' - \dot{q}''_{\min}}{\dot{q}''_{\max} - \dot{q}''} \right) \quad (3)$$

If, in a particular series of experiments, surface-catalyzed atom recombination alone were responsible for calorimeter heat fluxes falling between  $\dot{q}''_{\max}$  and  $\dot{q}''_{\min}$ , and if the assumptions underlying Eq. (3) were applicable, then a single ("best") value of the recombination coefficient  $\gamma$  should adequately correlate all of the heat flux data for a given calorimeter and surface temperature

#### Application to Air Arc-Tunnel Data

Both oxygen and nitrogen atoms are, in general, present in air arc-tunnel experiments. In accord with Eq. (1), allowance should therefore be made for expected differences in the

efficiency with which a particular calorimeter surface (selectively) catalyzes the recombination of these atoms. Although numerically less important, the effective recovery factors for chemical energy may be different as well. Thus, only in the special case of equal  $r_{D,i}$  and comparable recombination efficiencies<sup>††</sup> would Eq. (3) be applicable to air arc-tunnel experiments carried out in the presence of thermally significant amounts of atomic nitrogen. For example, in Ref. 1, the only experiments in which the effects of nitrogen dissociation can be neglected were those at 0.42 and 0.68 atm. At 0.136 atm, with  $h = 8800$  Btu/lbm, nitrogen atoms accounted for something like 70% of the freestream dissociation enthalpy. Nevertheless, if one formally subjects all of Wethern's published data<sup>‡‡</sup> to Eq. (3), one finds that the copper calorimeter was not strictly noncatalytic but, more likely, was characterized by recombination efficiencies of the order of  $10^{-3}$  at  $T_w = 366^\circ\text{K}$ . It is interesting to note that, under the highest pressure conditions investigated ( $p = 0.68$  atm), a calorimeter surface with a recombination coefficient of the order of  $10^{-1}$  would have, essentially, acted

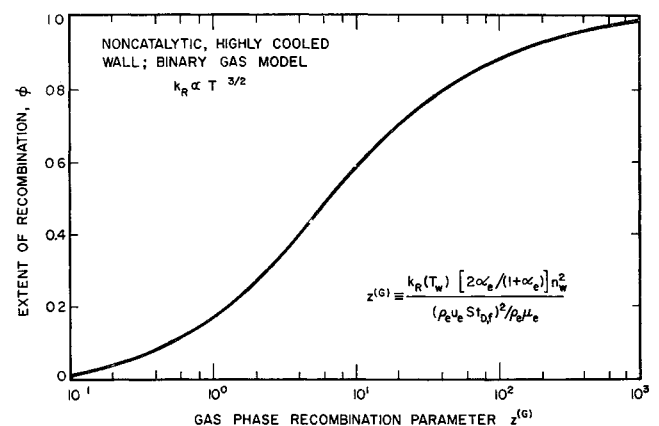


Fig. 1 Dependence of the extent of recombination  $\phi$  on the generalized recombination parameter  $z^{(G)}$  for boundary layers with negligible viscous dissipation

as a "perfect" catalyst, particularly in view of the precision of arcjet heat-transfer measurements

#### Gas-Phase Recombination

Before considering the implications of recombination probabilities of this order of magnitude, it is of interest to assess quantitatively the possible role of gas-phase atom recombination in determining the observed heat fluxes. Explicit neglect of gas-phase recombination, as done in the foregoing, evidently leads to an upper bound to the catalytic efficiency of the calorimeter surface. Indeed, it is, a priori, possible that the calorimeter surface was truly noncatalytic and that the observed heat flux difference  $\dot{q}'' - \dot{q}''_{\min}$  was due solely to atom recombination within the boundary layer. Let us examine this possibility for the highest pressure data point reported by Wethern,<sup>1</sup> since, in this experiment, 1) gas-phase recombination effects should have been most noticeable, and 2) only atomic oxygen was present at the

†† That is recombination coefficients that differ only by the square root of the atomic mass ratio

‡‡ It would appear that the constant  $0.793 (Pr_w)^{-0.6}$  given in Eq. (2) of Ref. 1 should be replaced by  $0.763 (Pr_w)^{-0.6}$ . Moreover, in calculating  $\dot{q}''_{\min}$  from the frozen flow form of Eq. (2) of Ref. 1, the nonunity Lewis number correction term should be dropped, since convection conduction is the only operative energy transfer mechanism in this limiting case.

§§ Larger values of  $\gamma$  would be indicated only if  $\dot{q}''_{\max}$  and  $\dot{q}''_{\min}$  were systematically overestimated, as can be seen by performing an error analysis using Eq. (3).

† Strictly speaking, the dependence of each recovery factor  $r_{D,i}$  on the Lewis number  $Le_i$  is specific to a given flow situation. However, in most cases of forced convection in dilute mixtures, departures from the simple approximation  $r_{D,i} \approx (Le_i)^{2/3}$  are small. In the nondilute case, each  $Le_i$  will vary appreciably across the boundary layer, and the recovery factor becomes dependent upon the extent of recombination and freestream species concentration.

§ This expression, which is exact at a stagnation point,<sup>2,4</sup> provides a good approximation elsewhere<sup>3</sup> when each  $z_i^{(w)}$  is based on the local species transfer coefficient  $St_{D,i}$ .

¶ In a binary gas, the equation of state implies  $\rho_w/\rho = (T/T_w)[(1 + \alpha_e)/(1 + \alpha_w)]$  where  $\alpha_w$  can be written as  $\alpha(1 - \phi)$ . For  $N \geq 2$ , however, an iterative procedure would be required to calculate the gas density adjacent to the surface.

\*\* In applying Eq. (3) to the data reported in Ref. 1, a useful relation, derivable from the heat-mass-transfer analogy for non-dissipative boundary layers ( $\frac{1}{2}u^2 \ll \Delta h_{\min}$ ) is  $\rho_w St_{D,i} = r_{D,i} \dot{q}''_{\min}/\Delta h_{\min}$ .

outer edge of the boundary layer (i.e., the binary gas approximation is valid) ¶¶ As suggested in Ref 6, for highly cooled surfaces the extent of the gas-phase reaction should be governed by a generalized recombination parameter of the form\*

$$z^{(G)} \equiv \frac{k_R(T_w) [2\alpha_e/(1 + \alpha_e)] n_w^2}{(\rho u St_D)^2 / \rho_e \mu_e} \quad (4)$$

This parameter incorporates all of the essential physical ideas discussed in Refs 5-8 and is readily evaluated from the data reported in Ref 1. However, an estimate of the termolecular rate constant  $k_R(T_w)$  must be introduced, and for this purpose we have used the result of Ref 9, extrapolated from 300° to 366°K on the basis of a  $T^{-3/2}$  power law. The value of  $(\dot{q}'' - \dot{q}''_{\min})/(\dot{q}''_{\max} - \dot{q}''_{\min})$  to be expected at each  $z^{(G)}$  for a noncatalytic surface can be estimated from the specific numerical stagnation-point† boundary-layer solutions of Fay and Riddell<sup>7</sup> by correlating their rate parameter with  $z^{(G)}$  in accord with the respective definitions and the methods outlined in Ref 6. The result is shown in Fig 1, which, for example, can be used to predict the pressure level at which gas-phase recombination alone would have led to the heat flux observed in experiment 7 of Ref 1. One finds that stagnation pressures in excess of 4 atm would have been required, implying that gas-phase recombination should play a minor role in determining calorimeter heat fluxes in sub-atmospheric arc-tunnels of the type investigated in Refs 1 and 10. This result also has important implications for materials testing in air arc-tunnels, particularly at higher specimen surface temperatures. For example, it is now known that the oxidation probability for O atoms can exceed that for O<sub>2</sub> molecules incident upon carbon,<sup>11</sup> platinum,<sup>12</sup> molybdenum,<sup>13</sup> and polyolefin<sup>14</sup> surfaces by more than two orders of magnitude. Thus the local oxygen atom concentration must be considered an important parameter in correlating flight and laboratory test data ‡

#### Catalytic Activity of Metal and Metal Oxide Surfaces

Values of  $\gamma$  of the order of  $10^{-3}$  might at first sight seem surprisingly low in view of some available atom recombination data for both copper<sup>15, 16, 20</sup> and copper oxide<sup>17, 18</sup> surfaces at temperatures near 300°K. However, it is unlikely that any kinetic data exist which overlap arc-tunnel conditions with regard to the detailed state of the test surface. Even in the absence of electrode impurities, the reasons for this are at least twofold. First, the identity and populations of the adsorbed species and even the nature of the effective adsorbent lattice are expected to be different for metals exposed to partially dissociated air at these pressures from, say, the bulk oxide exposed only to a mixture of oxygen atoms

and oxygen molecules, or "clean" copper exposed only¶ to a mixture of nitrogen atoms and nitrogen molecules. Second, the total pressures are very different, and there is evidence that increasing total pressure drastically reduces nitrogen atom recombination coefficients\*\* on cool copper and copper oxide surfaces,<sup>19</sup> perhaps because of the interference of weakly adsorbed gas layers. In short, it is probably overly optimistic to expect O-atom and N-atom recombination coefficients taken from independent studies at different pressures to apply under the composite conditions likely to be met in aerodynamic practice. This situation, which is in need of clarification, should be kept in mind in comparing future test data of the type quantitatively discussed here<sup>1</sup> with kinetic data obtained from other sources<sup>15-20</sup>

#### References

- Wethern, R. J., "Method of analyzing laminar air arc-tunnel heat transfer data," AIAA J 1, 1665-1666 (1963)
- Rosner, D. E., "Similitude treatment of hypersonic stagnation heat transfer," ARS J 29, 215-216 (1959)
- Rosner, D. E., "Convective heat transfer with chemical reaction—I. Theoretical development of correlation formulae for the prediction of heat fluxes in high performance rocket motors and related systems," Armed Services Technical Information Agency ASTIA AD 269 816, Aeronautical Research Lab Rept ARL 99, Part I (August 1961)
- Goulard, R. J., "On catalytic recombination rates in hypersonic stagnation heat transfer," Jet Propulsion 28, 737-745 (1958)
- Inger, G. R., "Highly nonequilibrium boundary layer flows of a multicomponent dissociated gas mixture," Aerospace Corp Rept SSD-TDR-63 253 (October 25, 1963)
- Rosner, D. E., "Scale effects and correlations in nonequilibrium convective heat transfer," AIAA J 1, 1550-1555 (1963)
- Fay, J. A. and Riddell, F. R., "Theory of stagnation point heat transfer in dissociated air," J Aeronaut Sci 25, 73 (1958)
- Inger, G. R., "Correlation of surface temperature effect on nonequilibrium heat transfer," ARS J 32, 1743-1744 (1962)
- Morgan, J. E., Elias, L., and Schiff, H. I., "Recombination of oxygen atoms in the absence of O<sub>2</sub>," J Chem Phys 33, 930-931 (1960)
- Winkler, E. L. and Griffin, R. N., Jr., "Effects of surface recombination on heat transfer to bodies in a high enthalpy stream of partially dissociated nitrogen," NASA TN-D-1146 (December 1961)
- Streznewski, J. and Turkevich, J., "The reaction of carbon with oxygen atoms," *Proceedings of the Third Symposium on Carbon* (Pergamon Press, New York, 1959), pp 273-278
- Fryberg, G. C., "Enhanced oxidation of platinum in activated oxygen," J Chem Phys 24, 175-180 (1956)
- Rosner, D. E. and Allendorf, H. D., "Discussion of: 'Oxidation of molybdenum, 550°C to 1700°C,'" AeroChem TP-75 (December 1963); also J Electrochem Soc III (to be published June 1964); also "Kinetics of high temperature oxidation of molybdenum by dissociated oxygen," AeroChem TP-84 (February 19, 1964); also J Chem Phys (to be published June 1, 1964)
- Mantell, R. and Ormand, W. L., "Activation of plastic surfaces in a plasmajet," AeroChem TP-81 (February 10, 1964); also presented at the American Chemical Society, Div of Fuel Chemistry Symposium: Chemical Phenomena in Plasma, Philadelphia, Pa (April 9, 1964)
- Prok, G. M., "Nitrogen atom recombination on surfaces," Planetary Space Sci 3, 38 (1961); also "Effect of surface preparation and gas flow on nitrogen atom surface recombination," NASA TN D 1090 (September 1961)
- Hoenig, S. A., "A study of oxygen recombination on metallic surfaces by means of an atomic beam," Univ of California, Institute of Engineering Research, TR HE-150 173 (October 7, 1960)

¶¶ Even if nitrogen atoms were also present, in accord with the results of Ref 5, the pressure at which gas-phase recombination comes into play cannot be smaller than that predicted on the basis of the binary gas/noncatalytic wall model

\* Here the atom mole fraction  $2\alpha/(1 + \alpha)$  has been used to describe the composition dependence of  $z^{(G)}$  at appreciable dissociation levels

† When cast in terms of  $z^{(G)}$ , results for the extent of recombination (e.g., Fig 1) are expected to provide a useful approximation away from the stagnation point<sup>6</sup> for nonequilibrium boundary layers with negligible viscous dissipation. Even at the forward stagnation point, this technique provides nonequilibrium convective heat-transfer predictions for flight conditions and wall temperatures other than those specifically treated in Ref 7

‡ This conclusion applies to the use of oxyacetylene torch tests as well; e.g., at the mixture ratio giving maximum adiabatic flame temperature (at  $p = 1$  atm), the O-atom number density exceeds that for O<sub>2</sub> (cf Ref 22)

§ As an example, the data reported in Ref 18 indicate that  $\gamma$  for O atoms on bulk copper oxide is  $1.6 \times 10^{-1}$  at  $T_w = 366$  K,  $p \approx 3 \times 10^{-3}$  atm. For N atoms on copper, Ammann<sup>20</sup> reports a sharp decline in  $\gamma$  at temperatures below 500°K. Above this temperature  $\gamma$  assumes a plateau value of about 0.6

¶ Even in nitrogen arc-tunnel testing<sup>10</sup> cooled models that were at one time fabricated in air will have surfaces on which oxygen is strongly chemisorbed

\*\* It has been suggested<sup>20</sup> that the pressure effect reported in Ref 19 may, alternatively, be the result of departures from continuum diffusion at large Knudsen numbers. These data should be re examined in this light<sup>21</sup>

<sup>17</sup> Greaves, J C and Linnett, J W Recombination of atoms at surfaces Part 5—Oxygen atoms at oxide surfaces, *Trans Faraday Soc* **55**, 1346–1354 (1959)

<sup>18</sup> Prok, G M, 'Nitrogen and oxygen atom recombination at oxide surfaces and effect of a tesla discharge on recombination heat transfer, NASA TN D-1567 (January 1963)

<sup>19</sup> Young R A "Pressure dependence of the absolute catalytic efficiency of surfaces for removal of atomic nitrogen" *J Chem Phys* **34**, 1292–1294 (1961); also 'Measurements of the diffusion coefficient of atomic nitrogen in molecular nitrogen and the catalytic efficiency of silver and copper oxide surfaces, *J Chem Phys* **34**, 1295–1301 (1961)

<sup>20</sup> Ammann, H H, "Heterogeneous recombination and heat transfer with dissociated nitrogen" Ph D Dissertation, Purdue University (January 1964)

<sup>21</sup> Hamel, B B, 'A study of some mass transfer problems according to the kinetic theory of gases,' Ph D Dissertation Princeton University (May 1963)

<sup>22</sup> Edwards, H E, Smith, R W, Jr, and Brinkley, S R Jr, "The thermodynamics of combustion gases: temperatures and composition of the combustion products of oxyacetylene flames" U S Bureau of Mines Rept 4958 (March 1953)

## Laminar Wake with Arbitrary Initial Profiles

HARRIS GOLD\*

Avco Corporation, Wilmington, Mass

### Introduction

KUBOTA<sup>1,2</sup> has solved the problem of a two-dimensional laminar compressible wake with an arbitrary streamwise pressure gradient by applying an Oseen-type linearization. The initial velocity and enthalpy distributions were assumed to be delta functions. This analysis has been extended to the axisymmetric case and to include arbitrary initial velocity and enthalpy distributions.

### Analysis

It is assumed that the flow in the viscous wake is described by the boundary-layer equations. By introducing the following transformations

$$x(x^*) = \int_0^{x^*} \left( \frac{\rho^* U^*}{\rho_\infty^* U_\infty^*} \right)^{1-m} \frac{\mu_e^*}{\mu_\infty^*} \frac{dx^*}{d^*} \quad (1a)$$

$$\frac{y^{m+1}(x^*, y^*)}{(m+1)} = \frac{\rho_e^* U^*}{\rho_\infty^* U_\infty^*} (RC)^{(m+1)/2} \int_0^{y^*} \frac{\rho^*}{\rho^*} y^* \frac{dy^*}{d^{*m+1}} \quad (1b)$$

$$C = \frac{\rho_e^* \mu^*}{\rho^* \mu^*} = \text{Chapman-Rubens factor} \quad (1c)$$

= const

Received February 10, 1964. This work was partially supported by the U S Army Research Office and the Advanced Research Projects Agency, Department of Defense, under Contract No DA-04-495-ORD-33 with the California Institute of Technology; this research is a part of Project DEFENDER sponsored by the Advanced Research Projects partially supported by the U S Air Force Ballistics System Division, Air Force Systems Command under Contract AFO4(694)-239 with Avco Corporation, Research and Advanced Development Division; this research is part of the REST Project sponsored by the Air Force Ballistics System Division. The author wishes to acknowledge T Kubota of the California Institute of Technology for his helpful discussions.

\*Staff Scientist, Research and Advanced Development Division. Member AIAA.

the boundary-layer equations become

Continuity

$$\frac{\partial u y^m}{\partial x} + \frac{\partial v y^m}{\partial y} = 0 \quad (2a)$$

Momentum

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{U} \frac{dU}{dx} (h - u^2) + \frac{1}{y^m} \frac{\partial}{\partial y} \left[ \left( \frac{\Lambda y^*}{d^* y} \right)^{2m} y^m \frac{\partial u}{\partial y} \right] \quad (2b)$$

Energy

$$u \frac{\partial h}{\partial x} + v \frac{\partial h}{\partial y} = \frac{1}{y^m} \frac{\partial}{\partial y} \left[ \frac{1}{\sigma} \left( \frac{\Lambda y^*}{d^* y} \right)^{2m} y^m \frac{\partial u}{\partial y} \right] + (\gamma - 1) M^2 \left( \frac{\Lambda y^*}{d^* y} \right)^{2m} \left( \frac{\partial u}{\partial y} \right)^2 \quad (2c)$$

State

$$h(x, y) = \frac{h^*(x^*, y^*)}{h^*(x^*)} = \frac{\rho_e^*(x^*)}{\rho^*(x^*, y^*)} \quad (2d)$$

where

$$\left. \begin{aligned} u(x, y) &= u^*(x^*, y^*) / U^*(x^*) \\ R &= \rho_\infty^* U_\infty^* d^* / \mu_\infty^* \\ \Lambda &= [(\rho^* U^* / \rho_\infty^* U_\infty^*) RC]^{1/2} \\ \sigma &= \text{Prandtl number} = \text{const} \\ U^* &= U^*(x^*) \\ U &= U(x) \\ h^* &= h^*(x^*) \\ h &= h(x) \\ m \begin{cases} = 0 & \text{two-dimensional flow} \\ = 1 & \text{axisymmetric flow} \end{cases} \end{aligned} \right\} \quad (3)$$

and the subscripts  $\infty$  and  $e$  refer to the freestream and local external quantities, respectively. The total local external enthalpy  $H^* = h^* + \frac{1}{2} U^{*2}$  is assumed to be constant. The boundary conditions are:

$$\left. \begin{aligned} u(0, y) &= u_0(y) & h(0, y) &= h_0(y) \\ u(x, \infty) &= 1 & h(x, \infty) &= 1 \\ m = 0 & \quad \partial u(x, 0) / \partial y = 0 & \quad \partial h(x, 0) / \partial y = 0 \\ m = 1 & \quad u(x, 0) \text{ finite} & \quad h(x, 0) \text{ finite} \end{aligned} \right\} \quad (4)$$

In the spirit of Oseen, let

$$\begin{aligned} u(x, y) &= 1 - \bar{u}(x, y) & \bar{u}(x, y) &\ll 1 \\ h(x, y) &= 1 + \bar{h}(x, y) & \bar{h}(x, y) &\ll 1 \end{aligned} \quad (5)$$

Substituting Eq (5) into Eqs (1) and (2), and retaining the lowest order terms, the following set of linear differential equations are obtained:

$$\begin{aligned} (\Lambda y^* / d^* y)^{2m} &\equiv 1 & m &= 0 \\ &= 1 + O(\bar{h}) & m &= 1 \end{aligned} \quad (6a)$$

$$v(x, y) = O(\bar{h}) \quad (6b)$$

$$\frac{\partial (U_e^2 \bar{u})}{\partial x} = \frac{1}{y^m} \frac{\partial}{\partial y} \left[ y^m \frac{\partial (U_e^2 \bar{u})}{\partial y} \right] - \frac{\bar{h}}{2} \frac{dU_e^2}{dx} \quad (6c)$$

$$\frac{\partial \bar{h}}{\partial x} = \frac{1}{\sigma} \frac{1}{y^m} \frac{\partial}{\partial y} \left( y^m \frac{\partial \bar{h}}{\partial y} \right) \quad (6d)$$

subject to the boundary conditions

$$\left. \begin{aligned} \bar{u}(0, y) &= \bar{u}_0(y) & \bar{h}(0, y) &= \bar{h}_0(y) \\ \bar{u}(x, \infty) &= 0 & \bar{h}(x, \infty) &= 0 \\ m = 0 & \quad \partial \bar{u}(x, 0) / \partial y = 0 & \quad \partial \bar{h}(x, 0) / \partial y = 0 \\ m = 1 & \quad \bar{u}(x, 0) \text{ finite} & \quad \bar{h}(x, 0) \text{ finite} \end{aligned} \right\} \quad (7)$$